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Boundary Condition at a Porous Surface Which Bounds a Fluid Flow

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A particular problem associated with the flow of a fluid over a porous surface is the selection of the appropriate boundary condition which must be used to give a realistic description of the tangential component of velocity at the permeable boundary. In order to allow for a nonzero velocity (that is, a slip velocity) at a porous boundary, Beavers and Joseph (1967) proposed a boundary condition in which the slip velocity is proportional to the velocity gradient at the boundary. The boundary condition involves an experimentally determined parameter which was argued to be independent of the fluid. Experiments with Poiseuille flows of oil and water through large aspect ratio rectangular ducts with one porous wall gave results which agreed with predictions based on the slip-flow boundary condition. Subsequent experiments with liquids in Poiseuille flows (Beavers et al., 1970; Sparrow et al., 1973) and Couette-type flow (Taylor, 1971) gave further support to the proposed boundary condition. Saffman (1971) has derived the form of the boundary condition analytically.

As yet, however, no investigations of the slip boundary condition in the presence of gas flows have been reported, and there has been no attempt to test the hypothesis that the slip parameter occurring in the boundary condition is independent of the fluid. The experiments described in this note were performed with the objective of establishing the validity of the slip boundary condition for gas flows and also of determining whether the fluid has a significant influence on the value of the slip parameter.

EXPERIMENTS

The experiments were performed in an open loop airflow facility. Air from a temperature-controlled laboratory room was drawn through a silica-gel dryer and a settling chamber into the test section. The test section was a large aspect ratio rectangular duct having a block of permeable material as the lower wall, an impermeable flat plate as the upper wall, and precisely machined spacer strips as the side walls. The smallest value of the duct height h was 0.24 mm (0.0097 in.) and the largest was 0.83 mm (0.0328 in.). The duct width was 8.9 cm (3.5 in.). Correspondingly, the cross-sectional aspect ratios ranged from about 110 to 360, thereby giving a close approximation to a parallel plate channel. The length of the test section was 40.6 cm (16 in.). The apparatus was designed so that the flows through the duct and the porous block were driven by the same axial pressure gradient.

Pressure gradients were recorded by means of taps which were axially distributed at 2.5-cm intervals along the center lines of both the upper wall of the duct and the lower wall of the test bed beneath the porous block. Pressure differences were read with a Baratron pressure gauge to as low as 0.01 mm mercury, and the measured pressure distributions along the upper and lower walls were found to be identical. The magnitude of the pressure gradient was chosen so that it was in the range for which a coupled parallel flow situation was established with fully-developed laminar flow in the channel and Darcy flow in the porous material. The two flows left the test section through separate exits, and each flow rate was measured by means of a rotameter.

The porous media consisted of two different specimens (Blocks A and B) of a material manufactured under the trade name of Foametal. This material consists of a homogeneous lattice-work of metal fibers such that there are no free fiber ends within the body of the material. Block A was 2.02 cm (0.795 in.) high and 32.5 cm (12.8 in.) long, while the corresponding dimensions for Block B were 2.25 cm (0.886 in.) and 37.5 (14.78 in.). The permeabilities were measured to be 6.67×10^{-7} cm^2 and 7.68×10^{-7} cm^2, respectively, for Blocks A and B. The porosities were on the order of 0.95, although they were not specifically measured for these experiments.

RESULTS

The slip condition at a permeable boundary is expressed by

$$(du/dy)_0 = (\alpha/k^{1/2})(u_0 - U) \qquad (1)$$

where α is the slip coefficient. The presence of a slip velocity affects both the friction factor and the mass flow in the duct. In the present experiments, measurements of friction factor (that is, pressure gradient) and mass flow were used to detect the effects of the slip velocity.

For fully-developed laminar flow in a parallel-plate channel with a slip boundary condition at one wall given by Equation (1), it can readily be shown (Beavers et al., 1970) that

$$\frac{C_f Re}{(C_f Re)^*} = \frac{m^*}{m} = \left[1 + \frac{3(\sigma + 2\alpha)}{\sigma(1 + \alpha\sigma)} \right]^{-1} \qquad (2)$$

where $\sigma = h/k^{1/2}$. Values of the $C_f Re$ product were calculated from the measured quantities for each experimental run, and the corresponding values of $(C_f Re)^*$ were computed from the theoretical solution for laminar flow in an impermeable-walled rectangular duct. The resulting values of the ratio $(C_f Re)/(C_f Re)^*$ for both porous blocks are plotted against Reynolds number for several values of $h/k^{1/2}$ in Figure 1. Inspection of Figure 1 shows that for a given $h/k^{1/2}$ there is no systematic dependence of $(C_f Re)/(C_f Re)^*$ with Reynolds number, which is consistent with the predictions of Equation (2). Further, since the data show that $(C_f Re)/(C_f Re)^* < 1$, there is a clear influence of velocity slip for all the duct heights used in these ex-

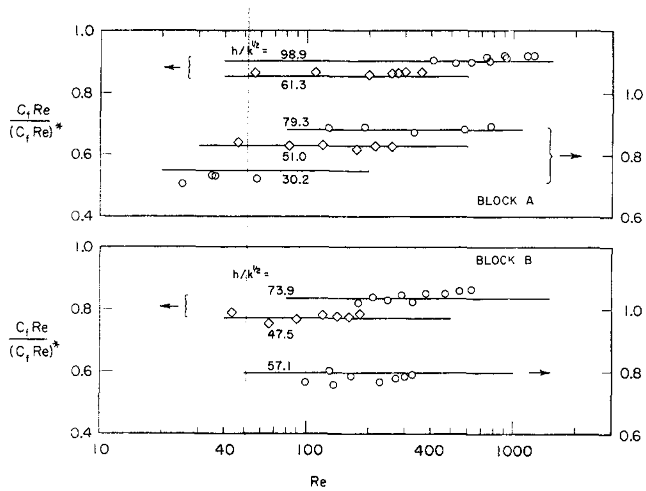

Fig. 1. Dimensionless friction factor, Reynolds number product as a function of duct Reynolds number.

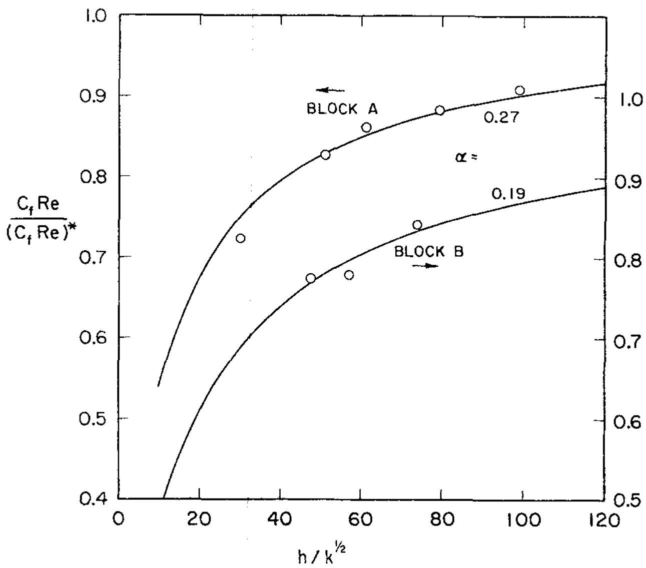

Fig. 2. Dimensionless friction factor, Reynolds number product as a function of dimensionless duct height.

periments, the effect becoming more pronounced as the height of the duct is decreased. For example, for the smallest duct height employed, there is a decrease of about 25% in the value of (C_fRe) from that for an impermeable-walled duct. A mean value of α for each block was computed from all the experimental data for that block. The straight line segments shown in Figure 1 are the predicted values obtained from Equation (2) using the mean α-values.

The variations of $(C_fRe)/(C_fRe)^*$ with $h/k^{1/2}$ as predicted from Equation (2) are shown in Figure 2. The predictions are compared with experimentally determined values, with each plotted point in Figure 2 representing the mean value over the range of Reynolds numbers at each duct height. The agreement is good, with the greatest deviation being about 4%.

These results demonstrate that a slip velocity at a porous boundary can be detected when the fluid flowing along the boundary is a gas. The results are also in agreement with predictions based on the slip boundary condition given by Equation (1) and thus provide further justification for the use of the boundary condition.

The second objective of this investigation was to compare the values of α obtained using air with the corresponding values obtained in earlier experiments using water as the working fluid. For Block A the values were 0.27 with air and 0.146 with water (Sparrow et al., 1973), while for Block B the values were 0.19 with air and 0.16 with water. There is an apparent large discrepancy between the two values for Block A. However, the air and water experiments with this block of porous material were conducted several years apart, and in the interim period between the two sets of experiments the surface of the material underwent a machining process to remove a small flaw that had developed. On the other hand, the water and air experiments with Block B were performed only a few weeks apart with the porous material in exactly the same state for both experiments. The agreement between the two values of α is much closer. For example, the difference between the predicted values of $(C_fRe)/(C_fRe)^*$ corresponding to α values of 0.19 and 0.16 is about 4% for $h/k^{1/2} = 47.5$ and about 3% for $h/k^{1/2} = 73.9$. This would indicate that the dependence, if any, of α upon the fluid may be small, whereas the experiments with Block A would indicate that the value of α is very sensitive to the nature of the porous surface. It appears that additional experiments would be valuable in order to substantiate conclusively the assertion of Beavers and Joseph (1967) that α depends only upon the permeable material and not upon the fluid.

ACKNOWLEDGMENT

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NOTATION

C_f = friction factor, $C_f = 2(-dp/dx)D/\rho\bar{u}^2$
D = hydraulic diameter of duct
h = duct height
k = permeability of porous material
m = mass flow rate through duct
$(-dp/dx)$ = streamwise pressure gradient
Re = Reynolds number, $Re = \bar{u}D/\nu$
U = superficial fluid velocity through porous material
u = fluid velocity in duct
$\bar{u}$ = mean fluid velocity in duct
y = transverse coordinate
α = slip parameter in Equation (1)
ν = kinematic viscosity
ρ = fluid density

Subscript

0 = conditions at the porous surface ($y = 0$)

Superscript

* = quantities applicable to an impermeable-walled duct

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